A THEORETICAL AND EXPERIMENTAL ANALYSIS OF LENGTHWISE PRESSURE GRADIENT FOR FLOW OF AIR IN SMALL BORE TUBING CONSIDERING THE EFFECT OF ELEVATED TEMPERATURE

A THESIS

Presented to

the Faculty of the Graduate Division

by

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Aeronautical Engineering

Georgia Institute of Technology

August, 1957

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A THEORETICAL AND EXPERIMENTAL ANALYSIS OF LENGTHWISE PRESSURE GRADIENT FOR FLOW OF AIR IN SMALL BORE TUBING CONSIDERING THE EFFECT OF ELEVATED TEMPERATURE

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Date Approved by Chairman August 28, 1957

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ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Doctor Arnold L. Ducoffe for suggesting the subject and giving invaluable assistance through all phases of the work. Gratitude is also extended to Doctor Robin B. Gray and Doctor Charles W. Gorton for their critical review of the topic, and to Messrs. J. Van Tassel, T. Long and F. De Jarnette (undergraduate students) for their assistance in running the experiment and reducing the data. Sincere appreciation is extended to the Sandia Corporation, whose financing of this project made this thesis possible.

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NOTATION

English

A	area of tubing, in^2
a	radius of tubing, ft^2
c	speed of sound, ft per sec
c_1	constant
c_2	constant
D	inside diameter of tubing, in
^oF	degrees Fahrenheit
g	acceleration of gravity, ft per sec^2
K	constant equal to (gRT)
k	ratio of specific heats equal to 1.4
L	length of tubing, ft
M	Mach number, V/c
M_e	exit Mach number
P	static pressure, psia
P_a	pressure downstream of the tube equal to atmospheric pressure, psia
P_e	static pressure at tube exit, psia
P_i	static pressure at tube entrance, psia

NOTATION (CONTINUED)

English

P_o	static pressure upstream of the tube, psia
Q	volume rate of flow, cu ft per sec
R	gas constant for air equal to 53.34 ft per degrees Rankine
r	pressure ratio, P_a/P_o
T	absolute static temperature, degrees Rankine
T_o	absolute static temperature upstream of the tube, degrees Rankine
$\bar{u}$	average velocity, ft per sec
V	velocity, ft per sec
w	flow rate, lbs per sec
x	lengthwise spatial coordinate

Greek

μ	viscosity, lb-sec per sq ft
π	3.1416
ρ	density, slugs per cu ft
Ω	flow factor, $\frac{w\sqrt{T_o}}{A\ P_o}$, (degrees Rankine)$^{1/2}$/sec

Subscripts

1	upstream spatial position
2	downstream spatial position
s	stagnation conditions upstream of tube

SUMMARY

This study presents an empirical method for determining the lengthwise pressure distribution in insulated small-bore tubing with special emphasis placed on elevated entrance temperature. It also explores the extent to which the empirical theory agrees with the experiment for subsonic air flow. The tubing tested, ranging in diameter from 0.180 to 0.524 inches and in length from 10 to 19.5 feet, was representative of that found in some gas lines, aircraft plumbing, and various other types of flow lines. The theoretical equation for the pressure distribution as a function of tube length was based on the assumption of (a) continuous medium, (b) constant area tubing, and (c) fully developed laminar flow over the entire length of tubing. With these assumptions the Hagen-Poiseuille law was used, with compressibility being introduced into the equations of motion by assuming isothermal changes of state in the fluid medium throughout the entire length of tubing.

The tubing was tested between sections of 4-inch extra heavy steel pipe in which the tube diameter/pipe diameter ratios ranged from 0.045 to 0.130. Changing the flow conditions was accomplished by adjusting the upstream temperature and pressure. The permanent pressure loss resulting from friction and end effects was determined by measuring the static pressure upstream and downstream of the test section. The static pressure distribution along the tubing was

measured through 0.020 inch holes drilled into the tubing. The study was limited to a maximum head pressure of 35 psig and a maximum head temperature of 500°F.

The theory approximates the pressure distribution in the tubing tested within 5 per cent up to an exit Mach number of 0.50 (approx.). In analyzing the lengthwise pressure distribution, the temperature effects are adequately accounted for in the definition of the flow factor, Ω. For the tube sizes investigated the entrance pressure ratio, P_i/P_o, and exit pressure ratio, P_e/P_a, were found to be only a function of the flow factor, Ω. Since the flow factor, Ω, is only a function of pressure ratio, r, for a given tube length and diameter, it was found that the pressure distribution was also only a function of the pressure ratio (r), tube length, and tube diameter.

CHAPTER I

INTRODUCTION

The subsonic flow of compressible fluids in constant area tubing is of importance in many engineering fields, including gas lines, aircraft and missile plumbing, and various other systems. This study is primarily concerned with presenting a method for determining the lengthwise pressure distribution in some small bore tubing with special emphasis placed on the effects of elevated temperature. Previous methods for determining the lengthwise pressure distribution (1) involved experimentally determining the amount of surface friction. In the present study an attempt is made to compare the experimentally measured pressure distribution for flow of air in small bore tubing with existing theory using experimentally measured boundary conditions at the tube entrance and exit.

The experimental data were taken from Bradley's work (2) in which the flow rate was found to be inversely proportional to the square root of the absolute temperature. Consequently the flow factor, Ω, was shown for various tubes to be only a function of pressure ratio (r), tube length, and tube diameter.

CHAPTER II

FLOW EQUATIONS

General.--The equations of motion are based on the assumption of a continuous medium, constant area tubing, and a fully developed laminar flow over the entire length of tubing. The change of state of the fluid medium throughout the entire length of tubing is assumed to be isothermal.

Continuum Flow.--In the development of the equation of motion for a constant area tube a fully developed laminar flow is assumed to exist over the entire length of tubing. This assumption fails in the vicinity of the tube entrance. However, for the tube lengths and diameters examined in the present tests, the effect of a relative short inlet length of undeveloped flow is considered negligible.

Starting with the Hagen-Poiseuille law (3, 4) for steady, incompressible, fully developed laminar flow, the volume rate of flow through the tubing is given as

$$Q = -\frac{\pi a^4}{8\mu} \frac{\partial P}{\partial x} = \pi a^2 \bar{u} \qquad (2.1)$$

where Q is the volume rate of flow, a is the inside radius of the tubing, P is the static pressure, x is the length spatial coordinate, μ is the coefficient of viscosity, and $\bar{u}$ is the average velocity at the cross-section of the tubing. Now, the rate of change of volume flow (4) between two stations Δx apart is given by

$$\frac{Q_2 - Q_1}{\Delta x} = \frac{-\pi a^4}{8\mu} \frac{(\partial P/\partial x)_2 - (\partial P/\partial x)_1}{\Delta x} \qquad (2.2)$$

Taking the limit as $\Delta x \rightarrow 0$:

$$\frac{\partial Q}{\partial x} = \frac{-\pi a^4}{8\mu} \frac{\partial^2 P}{\partial x^2} \qquad (2.3)$$

The steady, compressible continuity equation is written as

$$\frac{\partial \rho \bar{u}}{\partial x} = 0 \qquad (2.4)$$

where ρ is the fluid density. Substituting $\bar{u} = Q/\pi a^2$ into equation (2.4) results in

$$\frac{\partial \rho Q}{\partial x} = 0 \qquad (2.5)$$

It is assumed that the state changes in the insulated flow system take place by means of an isothermal process, that is,

$$P = K \rho \qquad (2.6)$$

where $K(= gRT)$ is constant. The constant g is the acceleration of gravity, R is the gas constant for air equal to 53.34 ft per degree Rankine, and T is static temperature. The assumption of an isothermal change of state is based on the following arguments. When the fluid is accelerated from a larger pipe or a reservoir the static temperature decreases. However, there is a static temperature rise in the fluid due to the viscous shearing stresses on the walls of the tubing. It is assumed that the drop in temperature due to acceleration and the rise in temperature due to viscous effects tend to counterbalance one another. Also, for low speed subsonic flow, the changes in static temperature are small.

Substituting equation (2.6) into the continuity equation, (2.5), results in

$$\frac{\partial PQ}{\partial x} = 0 \tag{2.7}$$

Performing the differentiation of equation (2.7) gives

$$P \frac{\partial Q}{\partial x} + Q \frac{\partial P}{\partial x} = 0 \tag{2.8}$$

Substituting equations (2.1) and (2.3) into equation (2.8), gives

$$P \frac{\partial^2 P}{\partial x^2} + \left(\frac{\partial P}{\partial x}\right)^2 = 0 \tag{2.9}$$

or

$$\frac{\partial^2 (P)^2}{\partial x^2} = 0 \tag{2.10}$$

A solution of equation (2.10) is

$$P(x) = \pm \sqrt{c_1 x + c_2} \tag{2.11}$$

The minus sign is dropped because the pressure is always positive. The constants c_1 and c_2 are evaluated by the boundary conditions:

$$\left.\begin{aligned} P &= P_i \quad \text{at} \quad x = 0 \\ P &= P_e \quad \text{at} \quad x = L \end{aligned}\right\} \tag{2.12}$$

and

where P_i is the entrance static pressure, P_e is the exit static pressure, and L is the tube length. Substituting equation (2.12) into equation (2.11) yields

$$P(x) = \sqrt{(P_e^2 - P_i^2)(x/L) + P_i^2} \tag{2.13}$$

or

$$\frac{P}{P_i} = \sqrt{\left[\left(\frac{P_e}{P_i}\right)^2 - 1\right] x/L + 1} \tag{2.14}$$

Equation (2.14) represents a viscous, compressible, isothermal, steady, continuous flow of a gas through tubes of constant area.

CHAPTER III

APPARATUS

As mentioned previously, the experimental data were taken from reference 2 and a complete description of the test apparatus can be found there. However, a brief description of the test section apparatus and its use is included. The test section is shown schematically in Fig. 1. The large pipe at each end of the test section was 4-inch extra heavy steel pipe (giving negligible approach velocity for the tubing tested). The test tubing was made of standard hard drawn copper. The diameters and lengths of the tubing tested are given in Table 1.

The blank flange on the pipe at the entrance end of the tube was tapped for a 3/4 inch pipe thread. The test specimen was connected to the flange by a close nipple, a reducer, and a straight-through compression fitting. In all cases the nipple, reducer, and compression fitting inside diameters were equal to or larger than the test tubing inside diameter. Thus the pressure drop through the fittings is negligible compared to the pressure drop through the tubing. The 4-inch pipe cap on the downstream end of the test section was tapped for a 1-inch pipe thread. The test tube exit end was connected to the pipe cap by a compression fitting, a reducer, a 1-inch gate valve, and 1-inch close nipple. Here again the inside diameter of all fittings was equal to or larger than the test tube

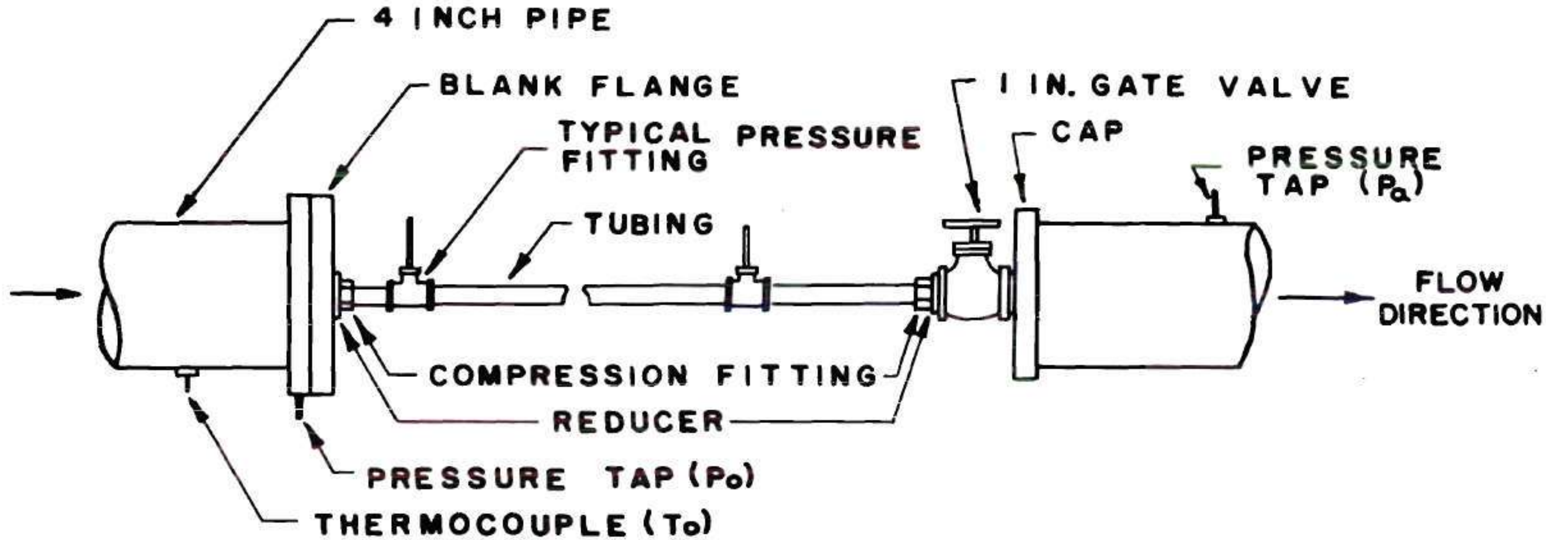

FIG. 1 TEST SECTION

Table 1. Tubing Tested

Length (ft)	Inside Diameter (in)
10	0.180
10	0.308
10	0.402
10	0.524
15	0.181
15	0.308
15	0.431
15	0.521
19.5	0.182
19.5	0.307
19.5	0.401
19.5	0.519

inside diameter. The gate valve was incorporated into the downstream side of the test section as a means for checking the system for leaks prior to each run.

For measuring static pressure at various positions along the tubing, a 0.020 inch hole was drilled into the tube at seven lengthwise positions. Then through a system of fittings and tubing the pressure was transmitted to manometers. The burrs on the inside of the tubing resulting from drilling were removed by polishing with emery cloth.

For the elevated temperature runs the air was heated prior to entering the test section. The heating sections and the test sections were insulated with 85 per cent magnesia high temperature insulation which would withstand temperatures up to 1500°F. An unshielded thermocouple, projecting into the stream and connected to a potentiometer, measured the static temperature ahead of the test section.

CHAPTER IV

PROCEDURE

A detailed explanation for the procedure used in obtaining the experimental data is given by Bradley (2). The method used to determine the flow rates was essentially that recommended by ASME (5, 6).

The experiments for each tube were undertaken for room temperature, 250°F (approx.), and 500°F (approx.). The high temperature was limited to 500°F because of the danger of pipe failure at the heater section. The maximum approach pressure was 35 psig.

CHAPTER V

RESULTS

General.--Equation (2.14) represents an approximation to the flow equation for the lengthwise pressure distribution under the conditions described. The boundary conditions for the solution of this equation were not measured because of the nature of the physical set-up of the experimental apparatus. However, an empirical method by which the boundary conditions can be obtained, if the reservoir pressure at each end of the tube, tube length, and tube diameter are known, is presented. Comparison of empirical theory and experiment is also included. Appendix B gives a problem as an illustration.

Boundary conditions.--The static pressure at the tube entrance, P_i, and exit, P_e, was not measured, but the pressure was measured several tube diameters from each end. At the same time the upstream static pressure, P_o, and downstream static pressure, P_a (which was atmospheric in every test), were measured in the 4-inch pipe. The entrance pressure in the test tubing is obtained by extrapolating the value of P/P_o (from figures of P/P_o vs x/L, represented typically by Fig. 2) to $x = 0$. The value of P at $x = 0$ is assumed to be P_i. Reference 2 shows that for a given tube length and diameter a flow factor, defined as

$$\Omega = \frac{w\sqrt{T_o}}{A\ P_o} \tag{5.1}$$

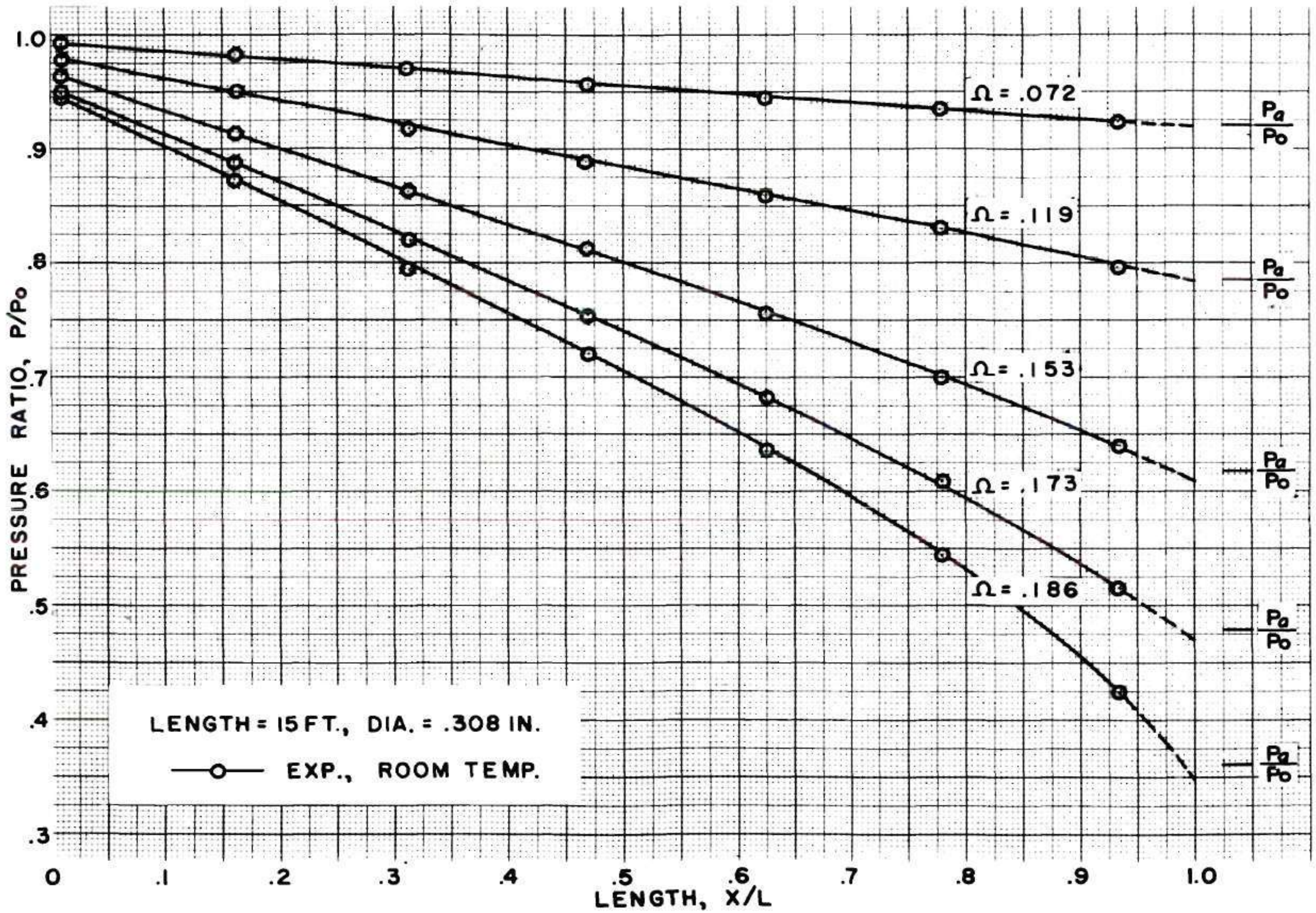

FIG. 2 PRESSURE RATIO VS. LENGTH

where w is the flow rate, T_o is the upstream static temperature, and A is the tube cross-sectional area, is a pure function of pressure ratio, r (= P_a/P_o). Fig. 3 represents a plot of P_i/P_o vs flow factor for the various lengths, diameters, pressure ratios, and temperatures tested. Some scatter in the data is present, yet it appears that a single curve will satisfy the variation of P_i/P_o as a function of Ω. Assuming that P_o and T_o are stagnation conditions, P_s and T_s, for the flow in the 4-inch pipe, included in Fig. 3 is a comparison of the empirical method for obtaining P_i with an analytical method, namely a compressible, isentropic perfect gas flow (Appendix C). From the comparison it is at once obvious that the losses because of the end effects cannot be predicted by elementary theory. The exit pressure ratio, P_e/P_a, is obtained by extrapolating curves of P/P_o to $x = L$ as shown in Fig. 2. Fig. 4 is the result of this extrapolation. Again some scatter of the data is present, but it is felt that a single curve adequately defines the variation of P_e/P_a as a function of Ω for the variations in tubing lengths and diameters, pressure ratios and temperatures employed in the present experiments. Assuming that the curves presented in Figs. 3 and 4 are adequate for defining P_i and P_e, the boundary conditions for the flow equation (2.14) are then determined once Ω is known for the particular geometry, dynamic, and thermodynamic test conditions. The lengthwise pressure distribution can now be calculated by use of equation (2.14). It should be noted that if experiments are conducted with the tube exit open to an infinite reservoir, Fig. 4 would not be needed because the

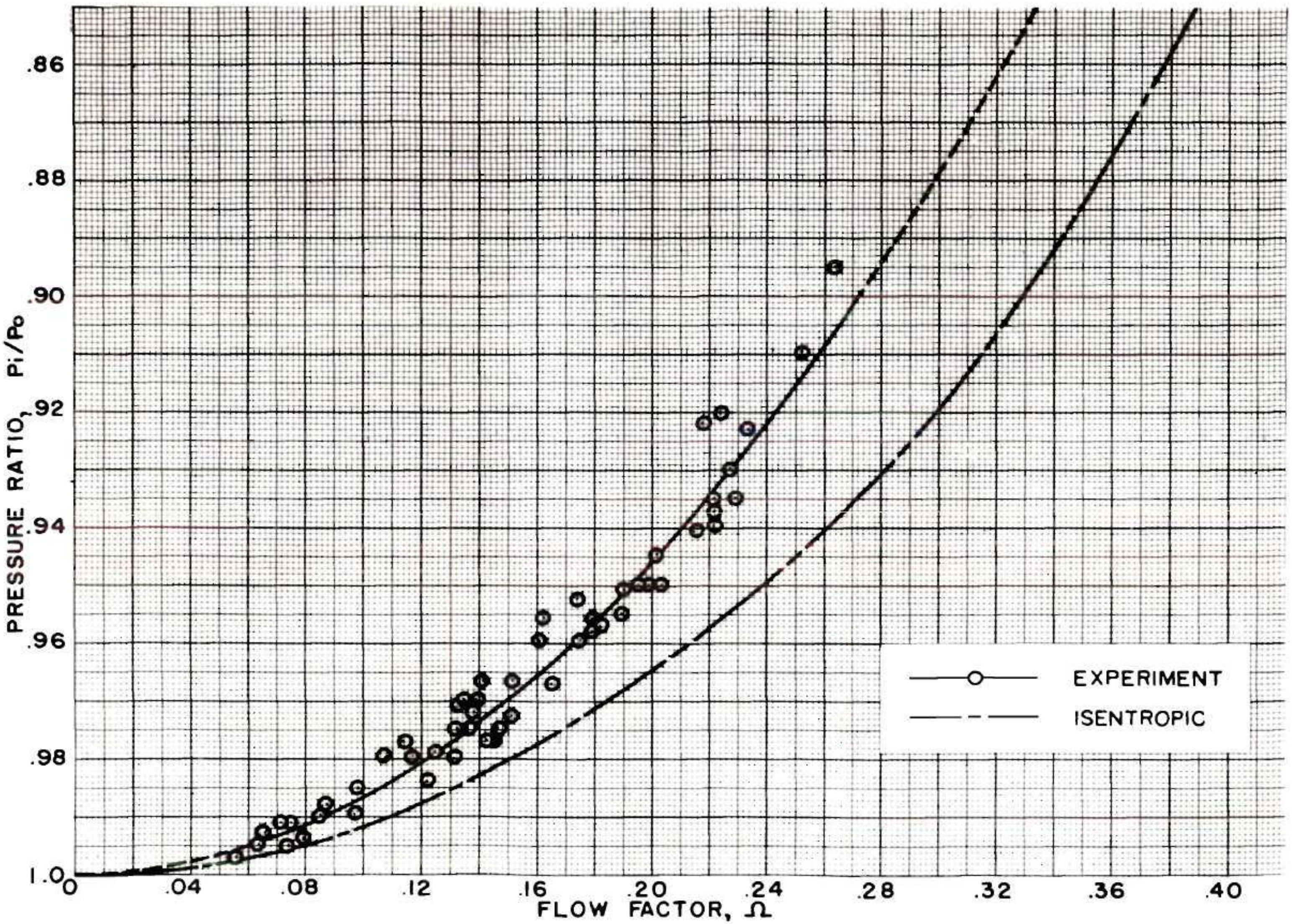

FIG. 3 ENTRANCE PRESSURE RATIO VS. FLOW FACTOR

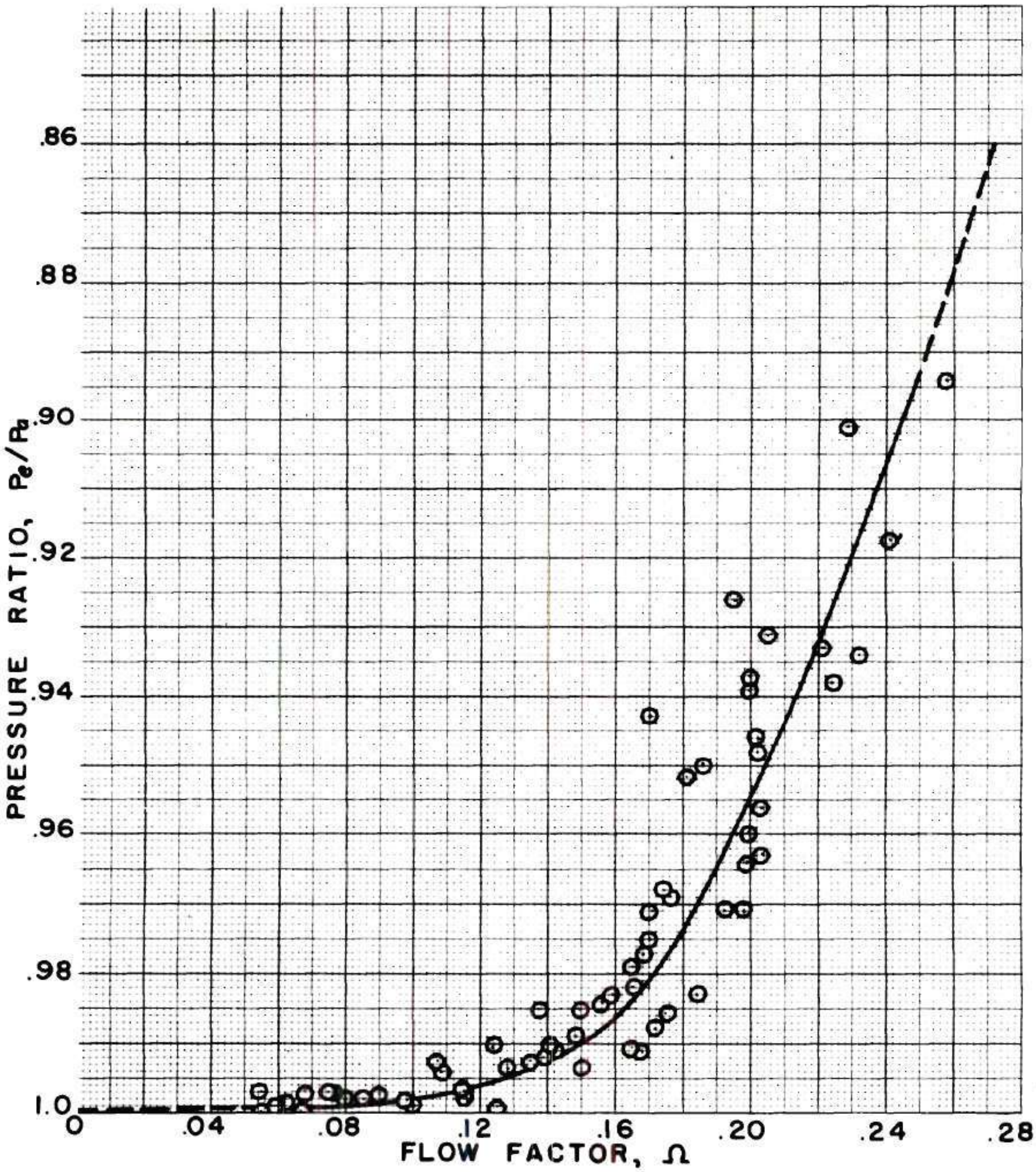

FIG. 4

EXIT PRESSURE RATIO VS FLOW FACTOR

static pressure at a subsonic jet exit is the same as the pressure in the infinite reservoir. Also, this figure is limited in its use, since it is valid only for conditions where air is being diffused over a range of diameter ratios (tube diameter/pipe diameter) from 0.045 to 0.130 (approximately).

Comparison of empirical theory and experiment.--Figs. 5 through 10 are representative plots showing the comparison of the empirical theory and experiment. The longest tube having the smallest diameter (Fig. 5) and shortest tube having the largest diameter (Fig. 6) are included to illustrate conditions of extreme L/D ratios tested where L is the tube length and D is the tube diameter. Figs. 7 through 10 were chosen at random from the other tubing tested. The experimental data were reduced from figures of P/P_o vs x/L, typically represented by Fig. 2. Then from Fig. 3 for the given flow factor, Ω , P_i/P_o were determined and the experimental data were reduced to figures of P/P_i vs x/L as shown in Figs. 5 through 10. These figures show that the agreement is very good at the low flow rates but the experimental data deviate from the empirical theory as the flow rate increases. It is also evident from Figs. 5 through 10 that temperature effects are adequately accounted for in the definition of the flow factor, Ω, since the experimental data for P/P_i at a given value of Ω are essentially coincident over the range of temperatures investigated. Since the pressure distributions shown in Figs. 5 through 10 are dependent only on Ω and the data from reference 2 shows Ω is only a function of P_a/P_o for a given tube length and diameter it appears that the pressure dis-

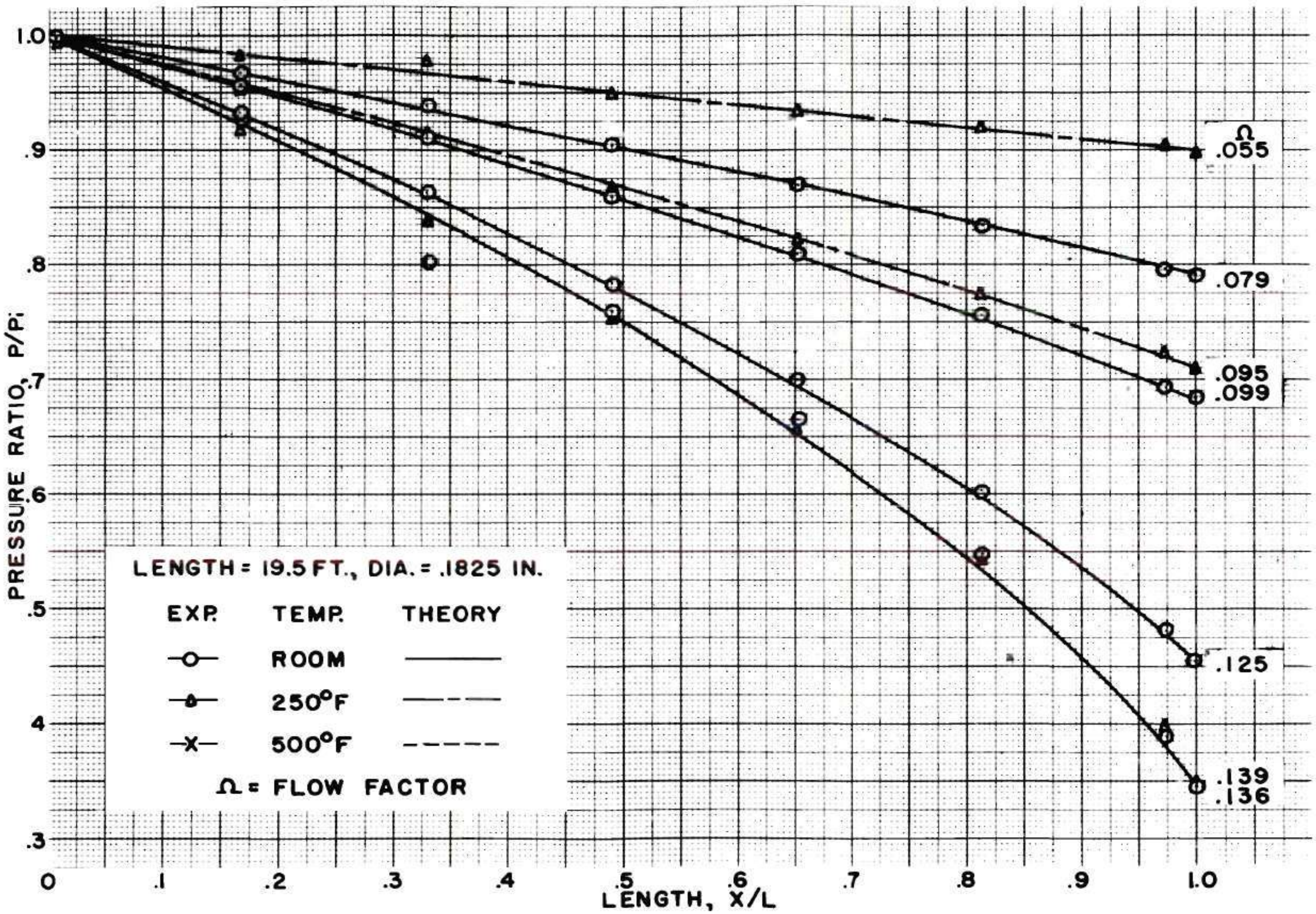

FIG. 5 PRESSURE RATIO VS. LENGTH

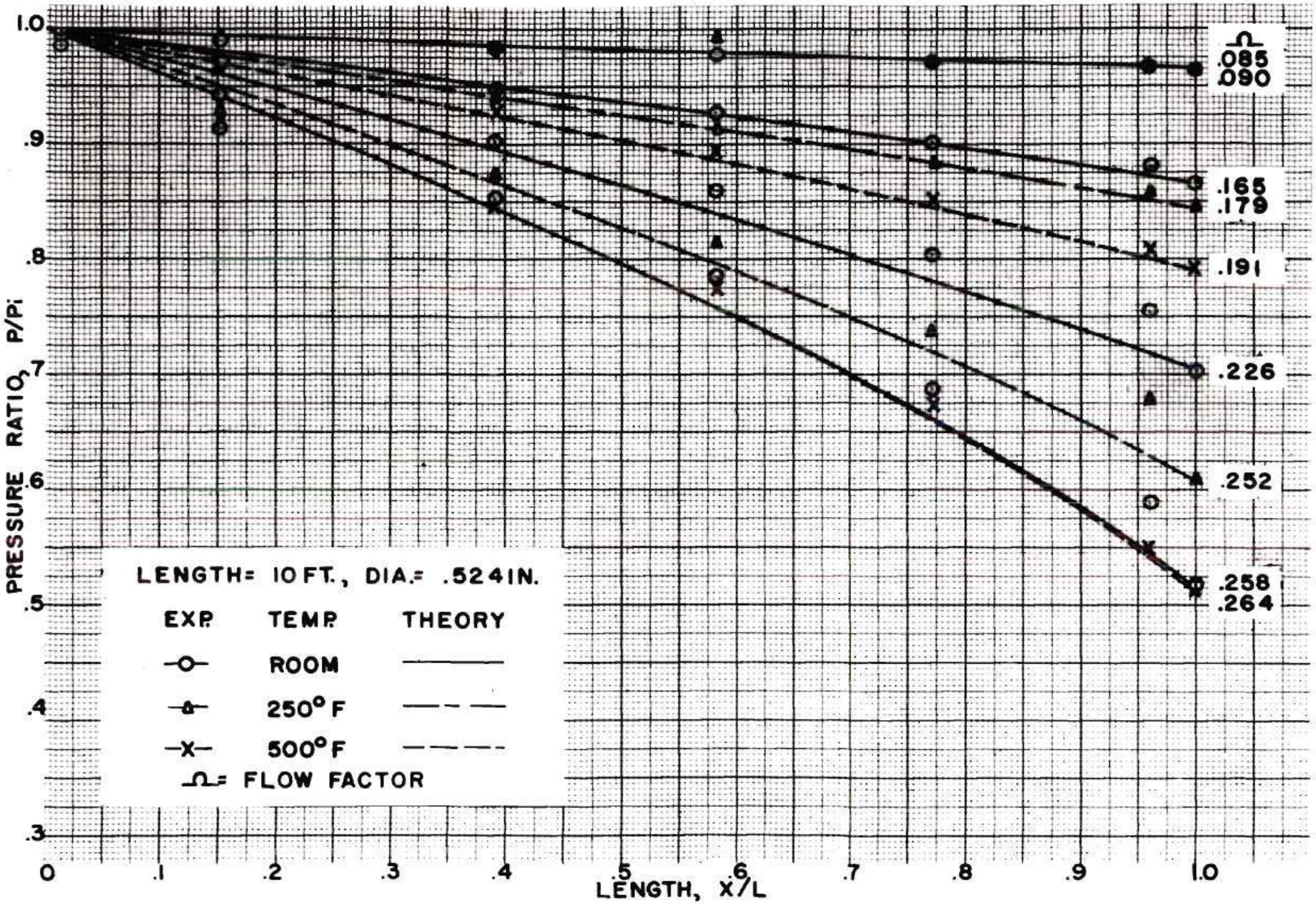

FIG. 6 PRESSURE RATIO VS. LENGTH

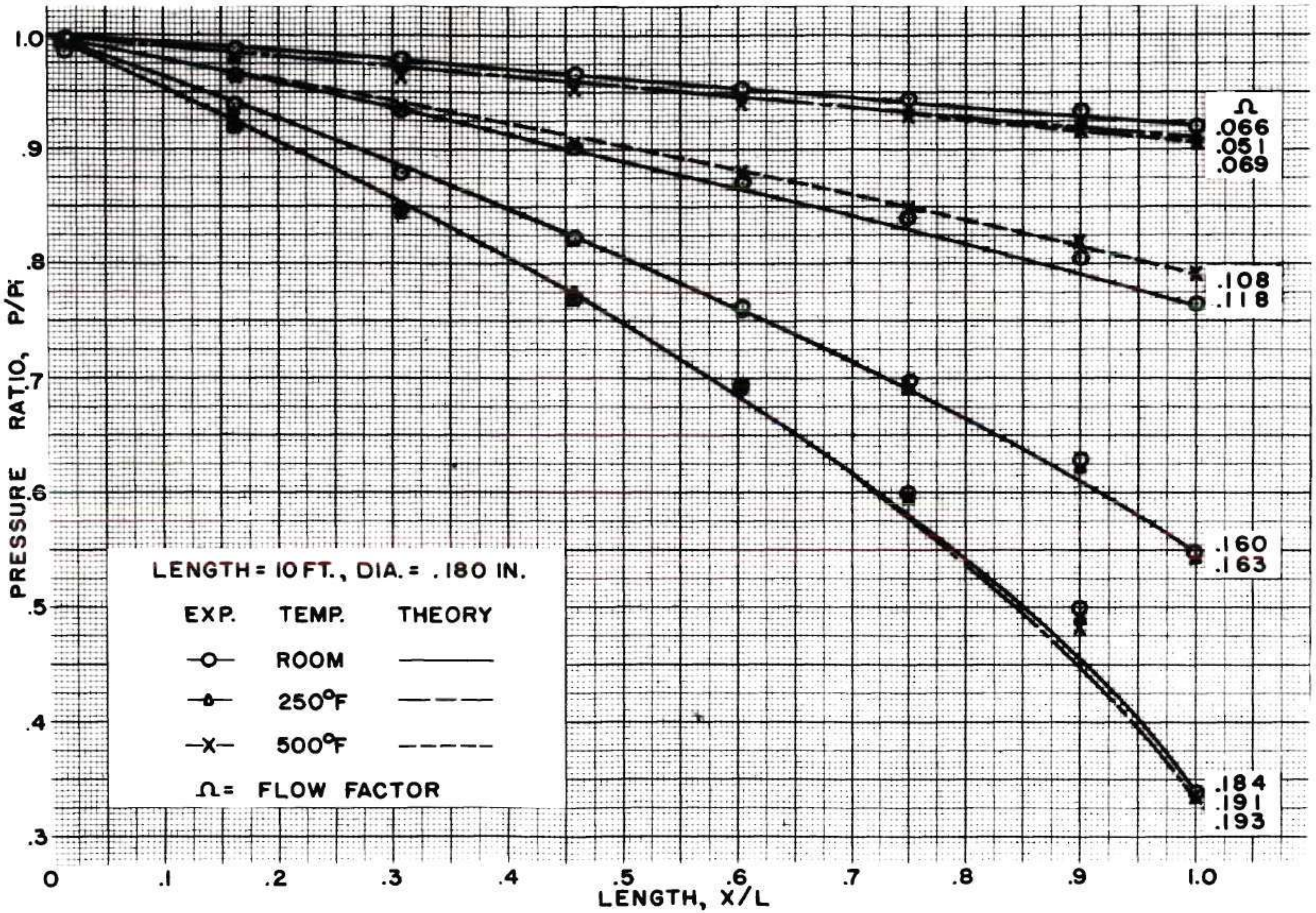

FIG. 7 PRESSURE RATIO VS. LENGTH

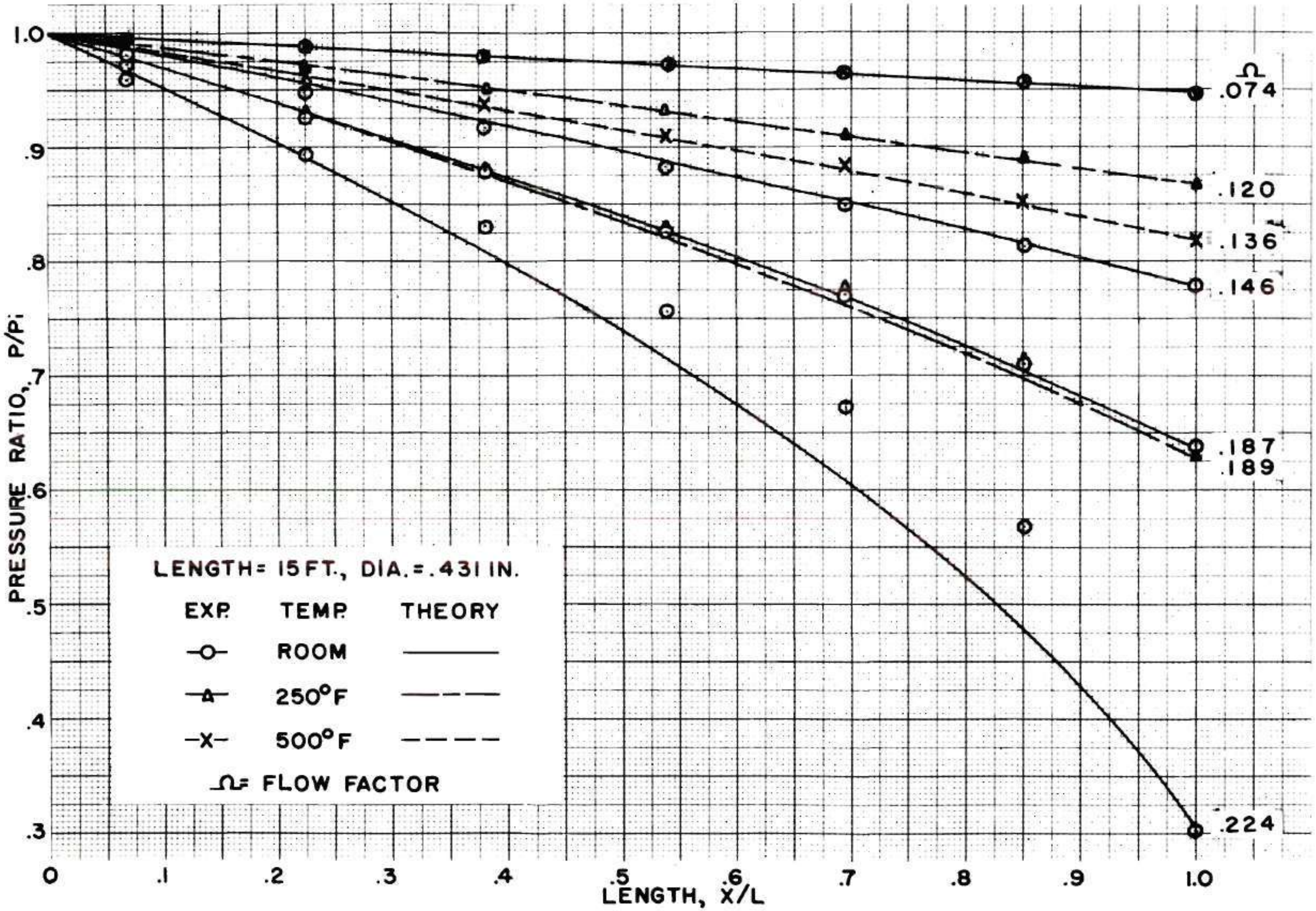

FIG. 8 PRESSURE RATIO VS. LENGTH

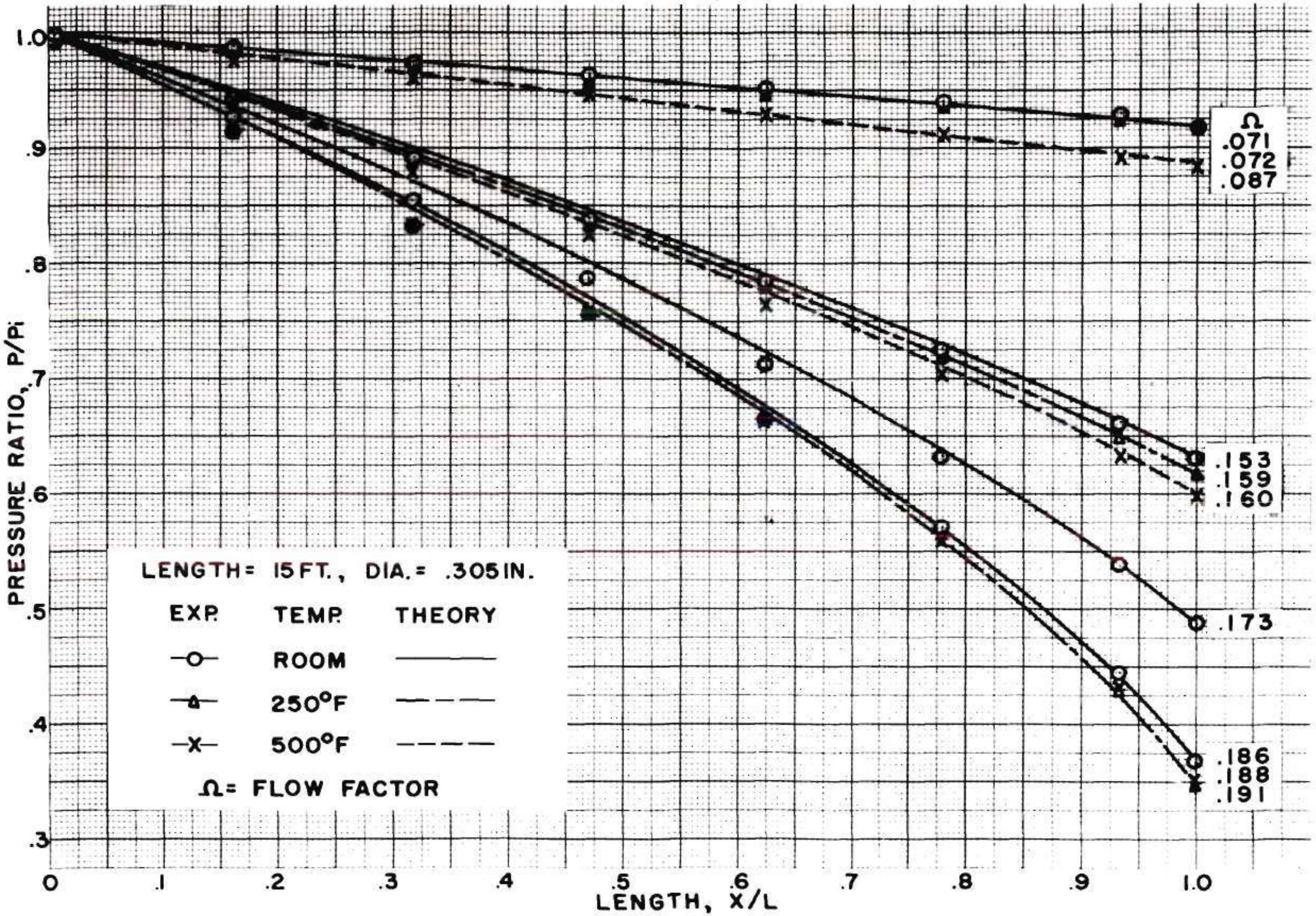

FIG. 9 PRESSURE RATIO VS. LENGTH

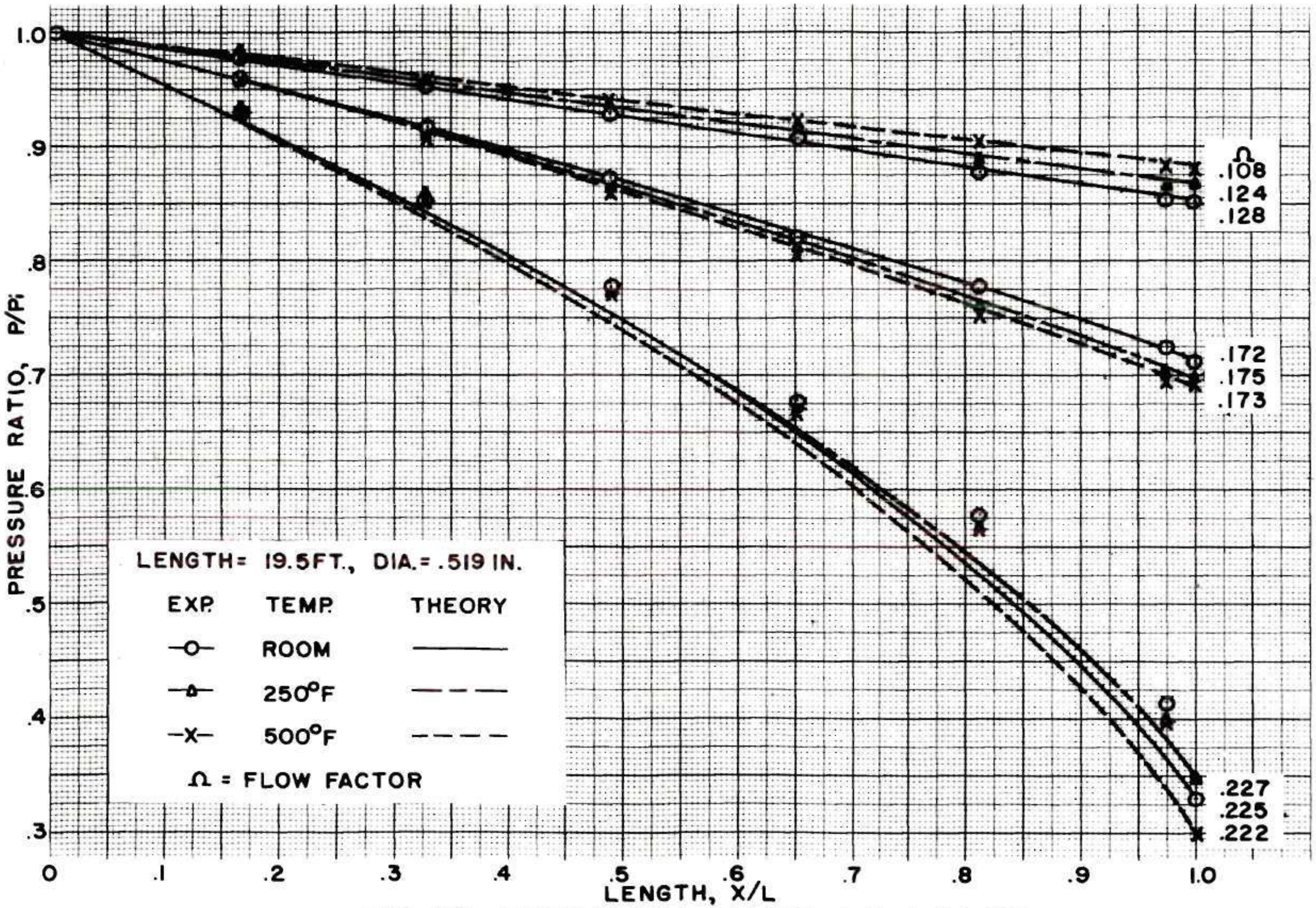

FIG. 10 PRESSURE RATIO VS. LENGTH

tribution can be described simply by the pressure ratio, r ($= P_a/P_o$), for a given geometry.

Assuming an isothermal flow of a perfect gas through the tubing, a Mach number (Appendix D) may be computed by

$$M(x) = \frac{P_o}{P} \; \Omega \sqrt{\frac{R}{k\,g}} \tag{5.2}$$

where M is the Mach number, P_o is the upstream static pressure, P is the static pressure at position, x, in the tubing, and k is the ratio of specific heats. If P_e is the static pressure at the tube exit, an exit Mach number may be computed by

$$M_e = \frac{P_o}{P_e} \; \Omega \sqrt{\frac{R}{k\,g}} \tag{5.3}$$

Using equation (5.3) to compute the exit Mach number, it was found that the theory represented by equation (2.14) agrees with experiment within 5 per cent up to an exit Mach number of 0.50 (approx.) for all tubing tested. Table 2 gives the exit Mach number for each tube tested for which the experiment was found to check the theory within 10 per cent. It is evident from Figs. 5 through 10 that the disagreement between theory and experiment becomes greater as x/L increases, since the Mach number in a constant area tube with friction increases as x/L increases if the inlet flow is subsonic. As Ω increases the disagreement is also greater because the Mach number is larger at each x/L position. It is felt that the disagreement between theory and experiment at the highest values of Ω may be due to choked flow.

Table 2. Approximate Exit Mach Number Limit in Which the Maximum Disagreement of Theory With Experiment is 10 Per Cent

Length (ft)	Diameter (in)	Mach No.
10	0.180	0.65
10	0.308	0.66
10	0.402	0.68
10	0.524	0.64
15	0.184	0.60*
15	0.308	0.63*
15	0.431	0.66
15	0.521	0.75
19.5	0.182	0.53*
19.5	0.307	0.67*
19.5	0.401	0.75
19.5	0.519	0.77

(*) Highest Mach number found and has better than 10 per cent agreement.

CHAPTER VI

CONCLUSIONS

This experiment was performed to simulate a practical and general method of creating a flow through small-bore tubing with negligible approach velocity. The fittings used on the tubing ends were standard and could be used to connect tubing to large pipes as used in this experiment or to a reservoir. The tubing tested ranged in diameter from 0.180 to 0.524 inches and lengths from 10 to 19.5 feet. The limitations on the experiment are maximum approach temperature of 500°F, maximum approach pressure or 35 psig, and diameter ratios (tube diameter/pipe diameter) from 0.045 to 0.130 (approx.). Within these limitations it may be concluded that:

1. The assumption of a steady, fully-developed, laminar flow with isothermal changes of state closely approximates the flow conditions in the tubing for which theory and empirical boundary conditions agree with experiment within 5 per cent up to exit Mach numbers of 0.50 (approx.).

2. In analyzing the lengthwise pressure distribution, the temperature effects are adequately accounted for in the definition of the flow factor, Ω .

3. The entrance pressure ratio, P_i/P_o, and exit pressure ratio, P_e/P_a , can be expressed as a function of flow factor, Ω , Figs. 3 and 4, respectively.

4. The lengthwise pressure distribution is dependent only upon pressure ratio, r (= P_a/P_o), tube length, and tube diameter.

CHAPTER VII

RECOMMENDATIONS

In a practical application it is possible that approach temperatures of 500°F would be exceeded and the diameter ratios (tube diameter/pipe diameter) would be different from 0.045 to 0.130. It is therefore recommended that a study be conducted to determine the flow characteristics of a system with higher temperatures and smaller and greater diameter ratios.

Tubes used in conducting air flow are not always in range of lengths of 10 to 19.5 feet and diameters of 0.180 to 0.524 inches. It is further recommended that a study of flow characteristics be extended to longer and shorter lengths and smaller and greater diameter tubing.

APPENDIX

APPENDIX A

FLOW FACTOR AS A FUNCTION OF PRESSURE RATIO AND TUBE DIAMETER FOR A GIVEN LENGTH

The flow factor is defined (2) as

$$\Omega = \frac{w\sqrt{T_o}}{A\,P_o} \tag{A.1}$$

where w is the flow rate, T_o is the upstream static temperature, P_o is the upstream static pressure, and A is the cross-sectional area of the tube tested. It was found that Ω is a pure function of the pressure ratio, r (= P_a/P_o), where P_a is the static pressure downstream of the test section. Fig. 11, taken from reference 2, is typical of flow factor as a function of pressure ratio, r, and tube diameter for a given length.

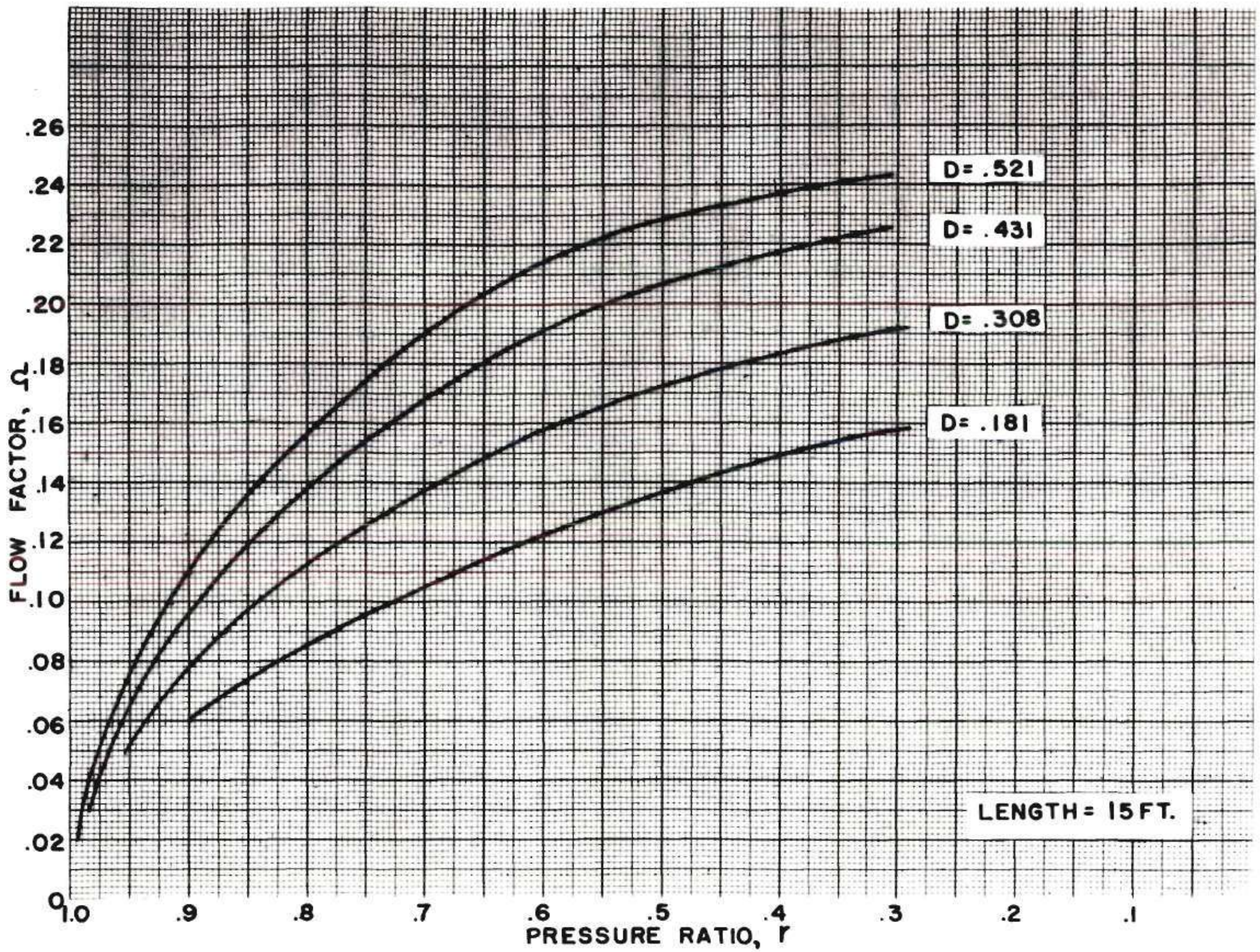

FIG. 11 FLOW FACTOR VS. PRESSURE RATIO

APPENDIX B

EXAMPLE PROBLEM: COMPUTING PRESSURE FOR A SPATIAL POSITION, GIVEN TUBE DIMENSIONS AND PRESSURE RATIO

Air enters a 15 foot tube with an inside diameter of 0.431 inches from a reservoir with a pressure of 23 psia. Find the pressure 7.5 feet from the tube entrance when (a) the air enters from and discharges into a 4 inch standard pipe that is open to the atmosphere with a pressure of 14.7 psia; (b) the air discharges from the tube directly into the atmosphere.

(a) The pressure ratio is

$$r = P_a/P_o = 0.639$$

From Fig. 11 $\quad \Omega = 0.183$

From Fig. 3 $\quad P_i/P_o = 0.954$

Therefore, $\quad P_i = 21.9$ psia

From Fig. 4 $\quad P_e/P_a = 0.971$

$$\frac{P_e}{P_i} = \frac{\left(\frac{P_e}{P_a}\right)\left(\frac{P_a}{P_o}\right)}{\left(\frac{P_i}{P_o}\right)} = 0.650$$

From equation (2.14)

$$\frac{P}{P_i} = \sqrt{\left[\left(\frac{P_e}{P_i}\right)^2 - 1\right]\frac{x}{L} + 1}$$

The pressure distribution is given as

$$\frac{P}{P_i} = \sqrt{1 - 0.578\,\frac{x}{L}}$$

Since $P_i = 21.7$ psia

and $\frac{x}{L} = 0.5$

$$P = 18.5 \text{ psia}$$

(b) Here $P_e = P_a$

Therefore, $$\frac{P_e}{P_i} = \frac{(P_a/P_o)}{(P_i/P_o)} = \frac{0.639}{0.954} = 0.670$$

The pressure distribution is

$$\frac{P}{P_i} = \sqrt{1 - 0.552\,\frac{x}{L}}$$

At $\frac{x}{L} = 0.5$ $\quad P = 18.8$ psia

APPENDIX C

FLOW FACTOR AS A FUNCTION OF STATIC PRESSURE AND STAGNATION PRESSURE FOR FLOW THROUGH A DUCT

This is a theoretical analysis for determining the ratio of the static pressure to the stagnation pressure given the flow factor. By assuming a compressible, perfect gas in which the flow is isentropic, Bernoulli's compressible flow equation gives

$$\frac{P}{\rho} = \frac{P_s}{\rho_s} - \frac{k-1}{k}\ \frac{V^2}{2} \tag{C.1}$$

and from the perfect gas law

$$\left.\begin{aligned} P &= \rho\, g\, RT \\ P_s &= \rho_s\, g\, RT_s \end{aligned}\right\} \tag{C.2}$$

where subscript, s, denotes stagnation conditions. From continuity, the flow rate, w, is given by

$$w = \rho\, g\, AV \tag{C.3}$$

and the flow factor is defined as

$$\Omega = \frac{w\sqrt{T_o}}{A\, P_o} \tag{C.4}$$

By assuming that P_o and T_o are stagnation conditions, P_s and T_s, the flow factor is

$$\Omega = \frac{w\sqrt{T_s}}{A\, P_s} \tag{C.5}$$

The isentropic relation between the pressure and density is

$$\frac{\rho}{\rho_s} = \left(\frac{P}{P_s}\right)^{\frac{1}{k}} \qquad \text{(C.6)}$$

Substituting the velocity from equation (C.3) into equation (C.1) gives

$$\frac{P}{P_s} = \frac{\rho}{\rho_s} - \frac{k-1}{2\,k}\,\frac{1}{\rho\,P_s}\left(\frac{w}{g\,A}\right)^2 \qquad \text{(C.7)}$$

Substituting for the density, ρ_s, from equation (C.2) into equation (C.5) yields

$$\rho = \frac{P_s}{g\,RT_s}\left(\frac{P}{P_s}\right)^{\frac{1}{k}} \qquad \text{(C.8)}$$

By substituting equations (C.5), (C.6), and (C.8) into equation (C.7) results in

$$\frac{P}{P_s} = \left(\frac{P}{P_s}\right)^{\frac{1}{k}} - \frac{R}{g\,R}\,\frac{k-1}{k}\left(\frac{P}{P_s}\right)^{-\frac{1}{k}}\Omega^2 \qquad \text{(C.9)}$$

or

$$\left(\frac{P}{P_s}\right)^{\frac{k+1}{k}} - \left(\frac{P}{P_s}\right)^{\frac{2}{k}} = -\frac{k-1}{2\,k}\,\frac{R}{g}\,\Omega^2 \qquad \text{(C.10)}$$

Equation (C.10) states that the ratio of the static pressure to the stagnation pressure is a function of only the flow factor in an isentropic flow of a perfect gas.

APPENDIX D

DETERMINATION OF THE MACH NUMBER FOR FLOW IN TUBING ASSUMING ISOTHERMAL CHANGES OF STATE

By assuming a perfect gas the equation of state is written as

$$P = \rho g R T \tag{D.1}$$

From continuity the rate of flow is given as

$$w = \rho g A V \tag{D.2}$$

The definition of the speed of sound is

$$c = \sqrt{k g R T} \tag{D.3}$$

or

$$V/M = \sqrt{k g R T} \tag{D.4}$$

where M is the Mach number. Substituting equation (D.4) into equation (D.2) gives

$$w = \rho g A M \sqrt{k g R T} \tag{D.5}$$

Substituting for the density from equation (D.1) yields

$$w = P A M \sqrt{\frac{k g}{R T}} \tag{D.6}$$

Assuming isothermal changes of state and applying equation (D.6) to the system in Fig. 12,

$$\frac{w \sqrt{T_o}}{A} = P M \sqrt{\frac{k g}{R}} \tag{D.7}$$

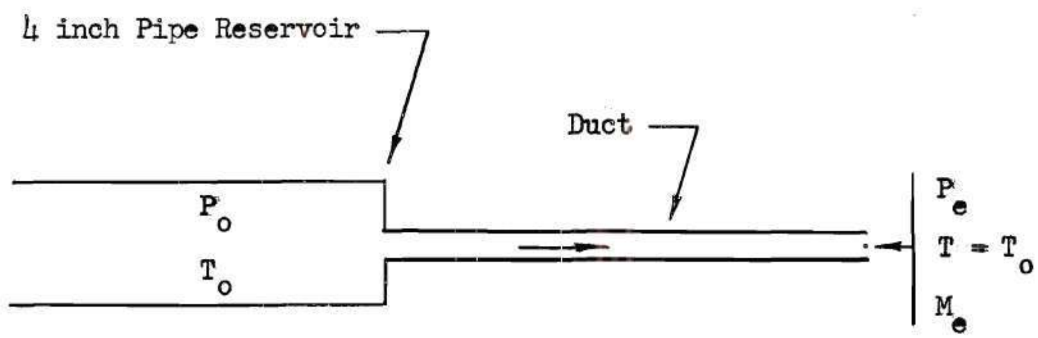

Fig. 12. Schematic Diagram for Isothermal Flow Through a Simple Duct

By the definition of the flow factor

$$\Omega = \frac{w \sqrt{T_o}}{A\, P_o} \tag{D.8}$$

equation (D.7) becomes

$$\Omega = \frac{P}{P_o}\, M \sqrt{\frac{k\, g}{R}} \tag{D.9}$$

or

$$M = \frac{P_o}{P}\, \Omega \sqrt{\frac{R}{k\, g}}$$

Equation (D.9) gives a Mach number at a lengthwise station in the tubing given the flow factor, local static pressure, and upstream static pressure. The exit Mach Number is given as

$$M_e = \frac{P_o}{P_e}\, \Omega \sqrt{\frac{R}{k\, g}} \tag{D.10}$$

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